

## *AP Calculus AB&BC Syllabus*

Certificated Teacher:

Date: 2015-2016

### Desired Results

**Course Title/Grade Level: AP Calculus**

**Credit: \_\_\_\_\_one semester (.5)                        X   two semesters (1)**

**Estimate of hours per week engaged in learning activities**

Students will typically commit a minimum of 5 hours of class work per week.

**Prerequisites and/or recommended preparation:**

Students should have completed PreCalculus/Integrated Math III and Algebra II prior to enrolling in AP Calculus.

**Instructional Materials:**

All learning activity resources and folders are contained within the student outline course. Online course is accessed via login and password assigned by student's school (web account) or emailed directly to student upon enrollment, with the login website address. Students then complete all assignments in MathXL. No additional textbooks or materials are required.

Note: All unit tests will be timed and taken individually in a proctored setting at approved testing locations with supervision - and must be completed with a TI-84 Calculator, no notes, and no other technology (including but not limited to internet browsers, cell phones, mp3 players, etc.). Additionally, students will have a maximum of 2 attempts on each unit test and must achieve a score of at least 60% on all prior assignments and quizzes before attempting the test.

**Course Overview:**

Students do best when they have an understanding of the conceptual underpinnings of calculus. This course stresses the dual concepts of conceptual understanding of calculus and fluency in the procedures that accompany those concepts. If students can grasp the reasons for an idea or theorem, they can usually figure out how to apply it to the problem at hand. We will study the following major ideas during the year: limits, derivatives, indefinite integrals, definite integrals, and Taylor series. Students practice the skills of calculus while they solve real-world problems with calculus concepts. Calculus is used everywhere from finding the volume and surface area of unusual shapes to understanding the behavior of vehicles and instantaneous rate of change.

**Enduring Understandings for Course:**

- Mathematics is a useful language for symbolically modeling and thus simplifying and analyzing our world.
- Mathematics is a logical and objective means of analyzing and solving problems.
- The effective communication of mathematics is essential to its application.
- The concept of a limit is one of the foundations of calculus.
- The derivative is the instantaneous rate of change at a given point.
- Differentiation and definite integration are inverse operations.
- There is a defined relationship between the integral of function  $f$ , the function  $f$ , and the first and second derivatives of function  $f$ .
- Integrals can be used to solve a variety of problems related to area, velocity, acceleration, volume, area of a surface of revolution, length of a curve, and work.

**List external resources and include cost for each.**

- Various free website resources including but not limited to: Wikipedia, Paul's Online Math Notes, Cool Schools, Visual Calculus, YouTube videos, various other online videos, MathXL (cost of \$15 per student paid for by the district), and Hofstra.

**Establish Goals:**

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; and Integrals as delineated in the Calculus AB Topic Outline in the *AP Calculus Course Description*

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

**Understandings:****Unit 1—Precalculus Review**

• Students focus on review of function concepts from precalculus that are a foundation for the calculus concepts they will learn to include:

- Exponential functions
- Parametric Equations
- Functions & Logarithms
- Trigonometric functions

**Unit 2—Limits & Continuity**

- The concept of a limit is one of the foundations of calculus.
- The limit of a function is the value approached by  $f(x)$  as  $x$  approaches a given value or infinity.
  - There is a connection between having a limit and being continuous in a function.
- There are a multiple ways of finding a limit by using a graph, numerically, or analytically.

**Unit 3—Derivative Basics**

- The derivative is the instantaneous rate of change at a given point.
- There is a connection between differentiability and continuity in a function.
- To take the derivative of a function there are a variety of rules that may be applied.
- There are connections between a function, its derivatives and their graphs.

**Unit 4—Derivatives with the Chain Rule**

- The derivative of a composite function can be found using the chain rule.
- The derivative of a variety of functions can be found by using a variety of other rules to include implicit, logarithmic, and by combining other derivative rules.

**Unit 5—Applying Derivatives****Essential Questions:****Unit 1—Precalculus Review**

- Can I graph and determine domain and range of a variety of functions like exponential, parametric equations, and trigonometric functions?
- Can I solve application & other kinds of problems involving these same functions?

**Unit 2—Limits & Continuity**

- What is a limit and how is it represented in a graph?
- How can the concept of limits be applied in mathematics?
- How would you find a limit using a graph, numerically, or analytically?
- How is the rate of change reflected in its table and graph?

**Unit 3—Derivative Basics**

- How does the derivative represent an instantaneous rate of change?
- How do you determine that a function is continuous and/or differentiable?
- Is there a way to visualize what a derivative is?
- How does the derivative of a function relate to its original equation?

**Unit 4—Derivatives with the Chain Rule**

- How do you take the derivative of a composite function?
- How do you take the derivative of a logarithmic or inverse function?

**Unit 5—Applying Derivatives**

- How are derivatives used in

- A derivative can be used to find the tangent line to a given function.
- Derivatives can be used to solve a variety of problems involving instantaneous rate of change and optimization.
- Derivatives can be used to understand the nature of a graph by finding extrema.
- Derivatives can be used to solve insolvable limit problems.

### **Unit 6—Basics of Integrals**

- Differentiation and definite integration are inverse operations.
- Integrals can be estimated using Riemann sums.
- Integrals are used to calculate area under a curve.
- Integrals are used to find antiderivatives.
- There is a defined relationship between the integral of function  $f$ , the function  $f$ , and the first and second derivatives of function  $f$ .

### **Unit 7—Solving Differential Equations**

- Explain the meaning of a differential equation and how to find solutions to them.
- Slope fields are used to find solutions to differential equations.
- Substitution and Integration by parts are techniques for integrating functions.
- Differential equations can be used to solve exponential and logistic problems.

### **Unit 8—Application of Integrals**

- Integrals can be used to solve a variety of problems related to area, velocity, acceleration, volume, area of a surface of revolution, length of a curve, and work.
- The integral is a function that can be used to determine the summation of an infinite set.

### **Unit 9—Series (BC students only)**

- There are series that are convergent and series that are divergent.
- There are a variety of different types of series to include geometric, harmonic, p-series, and alternating series.
- Information about a function and its derivatives can be determined from a Taylor polynomial.

### **Unit 10—Polar, Parametric, Vector Calculus (BC**

optimization problems?

- How do the graphs of the first and second derivatives relate to the function graph?
- How do you solve a limit problem when zero is divided by zero?
- How can a tangent line be used to approximate zeros of a function?

### **Unit 6—Basics of Integrals**

- How is the concept of a limit connected to a derivative and to an integral?
- How do you estimate an integral using Riemann sums?
- When given a function, how do you calculate area under a curve?
- What is an antiderivative and how are they used?
- What is the connection between a derivative and its antiderivative?

### **Unit 7—Solving Differential Equations**

- What is the meaning of a differential equation and how do you find solutions to them?
- What are slope fields and how do they solve differential equations?
- How do you use substitution and integration by parts to solve integrals?
- How are differential equations used to solve exponential and logistic problems?

### **Unit 8—Application of Integrals**

- How does the integral represent the summation of an infinite set?
- How are the following defined? (the area bounded by two curves, the volume generated by rotating a plane area, the length of a plane curve, the area of a surface revolution)
- What methods involving integrals can be used to find the volume of a solid?

### **Unit 9—Series (BC students only)**

- How do you write a series in sigma notation and determine its convergence?
- How do you tell the difference between geometric, harmonic, alternating, and p-series?
- What are power series and how do you identify common Maclaurin series?

### **Unit 10—Polar, Parametric, Vector**

<p><b>students only)</b></p> <ul style="list-style-type: none"> <li>• Students are introduced to the concept of a vector and its magnitude, direction and how it connects to velocity and acceleration.</li> <li>• Polar coordinates is another way to view graphing as opposed to the Cartesian system.</li> <li>• Technology can be used to graph and represent parametric, polar, &amp; vector functions.</li> </ul>	<p><b>Calculus (BC students only)</b></p> <p>Can you find a velocity vector and acceleration vector given the position vector?</p> <p>What is the connection between the Cartesian coordinate system and the polar coordinate system and how would you convert between them?</p>
<p><b>Students will know and be able to:</b></p> <p><b>Unit 1</b></p> <p>→Students will be able to graph exponential, parametric equations, logarithmic, and trigonometric functions.</p> <p>→Students will be able to solve application and various other exponential, parametric equations, logarithmic, and trigonometric problems.</p> <p><b>Unit 2</b></p> <p>→Students will understand the basics of Limits by explaining the (non)existence of a limit, recalling and applying properties of limits, and creating graphs/situations that fit given conditions.</p> <p>→Students will understand how to find limits graphically, numerically, analytically (substitution, simplification &amp; sandwich theorem) and limits involving infinity</p> <p>→Students will understand continuity and be able to recite/explain the definition of continuity, find/name points of discontinuity, and extend functions to be continuous</p> <p>→Students will understand rates of change and how to find instantaneous &amp; average rates of change, and be able to explain the difference between the two.</p> <p><b>Unit 3</b></p> <p>→Students will understand the definition for a derivative by explaining and using both forms of the definition, finding derivatives at a point, finding derivatives of functions, explaining points of non-differentiability, and by understanding instantaneous rates of change.</p> <p>→Students will find basic derivatives of algebraic (power rule), trigonometric, inverse trigonometric, logarithmic, and exponential type functions.</p> <p>→Students will understand derivatives of combined functions by using the product rule, quotient rule, and the chain rule.</p> <p>→Students will apply techniques to find derivatives by combining rules, using logarithmic differentiation, and by implicitly finding derivatives.</p> <p>→Students will be able to interpret derivatives by explaining the meaning of a derivative in context and by explaining the connections between a function and its higher-order derivatives</p> <p><b>Unit 4</b></p> <p>→Students will understand the derivative of a composite function can be found using the chain rule.</p> <p>→Students will use the derivative of a variety of functions can be found by using a variety of other rules to include implicit, logarithmic, and by combining other derivative rules.</p> <p><b>Unit 5</b></p> <p>→Students will understand tangent lines by finding the equation of a tangent line, using a tangent line to approximate a function (linearization), and by using a tangent line to approximate the zeros of a function (Newton's method).</p> <p>→Students will understand the mean value theorem by explaining the meaning of the MVT and by finding values guaranteed by the MVT.</p> <p>→Students will understand extrema by using a variety of techniques to find and justify them, and by optimizing a scenario.</p> <p>→Students will understand related rates by finding equations using related rates of change and by finding unknown rates of change in a scenario.</p> <p>→Students will understand L'Hopital's Rule by identifying indeterminate forms, using algebra to rewrite indeterminate forms in the 0/0 form, and by evaluating limits with the rule.</p> <p><b>Unit 6</b></p> <p>→Students will understand the concept of an integral by explaining the "net change" an integral finds, explaining what an integral calculates, by finding integral values for functions whose graphs are piece-</p>	

wise, simple geometry forms, and by recalling and applying properties of integrals.

→Students will estimate integrals by using various types of Riemann sums to estimate integral values and by using other methods to approximate integral values.

→Students will understand the mean value theorem for integrals by explaining the meaning of the MVT and by finding values guaranteed by the MVT.

→Students will know the fundamental theorem of calculus Part 1 (Area Accumulator) by finding values, graphing, and finding derivatives for area accumulator functions.

→ Students will know the fundamental theorem of calculus Part 2 (Evaluative form) by finding antiderivatives, and evaluating definite integrals with basic antiderivatives.

→Students will understand improper integrals by evaluating improper integrals with an infinite bound and with an infinite discontinuity.

### **Unit 7**

→Students will understand differential equations by explaining what it means to solve a differential equation, by finding solutions to basic differential equations (including +C), and by using initial values to find solutions.

→Students will understand slope fields by creating slope fields from a differential equation, by reading/interpreting slope fields, matching slope fields to differential equations, and by using slope fields to identify solutions to differential equations.

→Students will understand techniques for integration by integrating using substitution, integrate by parts, and by using partial fraction decomposition to integrate.

→Students will understand procedures to solve differential equations by separating variables in order to find solutions and by using Euler's method to approximate solutions.

→Students will understand applications of differential equations by solving exponential growth and decay problems and by solving logistic growth problems.

### **Unit 8**

→Students will understand integrals as net change by explaining the difference between displacement & distance traveled and by calculating net change in a variety of situations.

→Students will understand area between curves by setting up integrals, using  $dx$  &  $dy$  that represent the area between curves, and finding area between polar curves.

→Students will understand solids of known cross sections by finding volumes using the disc & washer methods, find volumes of solids not formed by rotation, and by using  $dx$  &  $dy$  cross sections.

→Students will understand lengths of curves by finding lengths of curves over a given domain.

### **Unit 9 (BC students only)**

→Students will understand series of functions by writing a series or partial sum from sigma notation, use notation to describe a series, and explain the difference between a convergent and divergent series.

→Students will understand series of constants by identifying & calculating geometric series, identifying harmonic series and convergence of alternating series. Students will also identify & determine convergence of p-series, will use the integral and comparison tests to determine convergence.

→Students will understand series of functions by identify/simplify power series, creating Taylor polynomials, finding information about a function & its derivatives from a Taylor polynomial, recalling/identifying common Maclaurin series, estimating errors of Taylor polynomials and by finding radius & interval of convergence (using ratio test).

### **Unit 10 (BC students only)**

→Students will understand the concept of a vector and its magnitude, direction and how it connects to velocity and acceleration.

→Students will explore Polar coordinates as another way to view graphing as opposed to the Cartesian system.

→Students will use Technology to graph and represent parametric, polar, & vector functions.

### **Review Unit**

→Students will prepare for the AP test by taking practice tests and evaluating their understanding and progress to correct and adjust their understanding and practice.

## **Evidence of Assessment**

### **Performance Tasks:**

#### **Unit 1**

→Students will complete 4 MathXL assignments and a final unit assessment.

#### **Unit 2**

→ Students will complete 4 MathXL assignments, 2 MathXL quizzes, and a final unit assessment.

**Unit 3**

→ Students will complete 5 MathXL assignments, 2 MathXL quizzes, and a final unit assessment.

**Unit 4**

→ Students will complete 4 MathXL assignments, 2 MathXL quizzes, and a final unit assessment.

**Unit 5**

→ Students will complete 6 MathXL assignments (7 for BC students), 2 MathXL quizzes, and a final unit assessment.

**Unit 6**

→ Students will complete 5 MathXL assignments (6 for BC students), 2 MathXL quizzes, and a final unit assessment.

**Unit 7**

→ Students will complete 5 MathXL assignments, 2 MathXL quizzes, and a final unit assessment.

**Unit 8**

→ Students will complete 4 MathXL assignments, 1 MathXL quiz, and a final unit assessment.

**Unit 9 (BC students only)**

→ Students will complete 6 MathXL assignments, 1 MathXL quiz, and a final unit assessment.

**Unit 10 (BC students only)**

→ Students will complete 3 MathXL assignments and a final unit assessment.

**Review Unit**

→ Students will complete up to four practice tests of the free response format and complete several online practice tests at a given website to prepare for the AP test. Students will then evaluate their understanding and progress to correct and adjust their understanding and practice.

**Other Evidence (self-assessments, observations, work samples, quizzes, tests and so on):**

Unit 1 through Unit 10: students complete a comprehensive unit test in each unit.

Other evidence that will be collected to show student understanding will be individual self-assessments for each unit as well as reflections on discussion board dialogue/questions with the class. There will be in-person proctoring for end of unit tests.

**Types of Learning Activities**

**Learning Activities:**

**Unit 1**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 4 MathXL assignments and a student reflection upon completion.

**Unit 2**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 4 MathXL assignments and a student reflection upon completion.

**Unit 3**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 5 MathXL assignments and a student reflection upon completion.

**Unit 4**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 4 MathXL assignments and a student reflection upon completion.

**Unit 5**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 6 or 7 MathXL assignments and a student reflection upon completion.

**Unit 6**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 5 or 6 MathXL assignments and a student reflection upon completion.

**Unit 7**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 5 MathXL assignments and a student reflection upon completion.

**Unit 8**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 4 MathXL assignments and a student reflection upon completion.

**Unit 9 (BC students only)**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 6 MathXL assignments and a student reflection upon completion.

**Unit 10 (BC students only)**

→ Students will explore a powerpoint presentation, online resources; observe online web videos explaining the concepts (listed in external resources above) and then complete 3 MathXL assignments and a student reflection upon completion.

**Review Unit**

→ Students will complete up to four practice tests of the free response format and complete several online practice tests at a given website to prepare for the AP test. Students will then evaluate their understanding and progress to correct and adjust their understanding and practice.

Indicate from the table below all applicable learning strategies that may be used in the course.

Direct Instruction	Indirect Instruction	Experiential Learning	Independent Study	Interactive Instruction
<input type="checkbox"/> Structured Overview <input checked="" type="checkbox"/> Mini presentation <input checked="" type="checkbox"/> Drill & Practice <input checked="" type="checkbox"/> Demonstrations <input type="checkbox"/> Other (List)	<input checked="" type="checkbox"/> Problem-based <input checked="" type="checkbox"/> Case Studies <input checked="" type="checkbox"/> Inquiry <input type="checkbox"/> Reflective Practice <input type="checkbox"/> Project <input type="checkbox"/> Paper <input type="checkbox"/> Concept Mapping <input type="checkbox"/> Other (List)	<input type="checkbox"/> Virt. Field Trip <input checked="" type="checkbox"/> Experiments <input type="checkbox"/> Simulations <input type="checkbox"/> Games <input type="checkbox"/> Field Observ. <input type="checkbox"/> Role-playing <input type="checkbox"/> Model Bldg. <input type="checkbox"/> Surveys <input type="checkbox"/> Other (List)	<input type="checkbox"/> Essays <input checked="" type="checkbox"/> Self-paced <input type="checkbox"/> computer Journals <input type="checkbox"/> Learning Logs <input type="checkbox"/> Reports <input type="checkbox"/> Directed Study <input type="checkbox"/> Research <input type="checkbox"/> Projects <input type="checkbox"/> Other (List)	<input checked="" type="checkbox"/> Discussion <input type="checkbox"/> Debates <input type="checkbox"/> Role Playing <input type="checkbox"/> Panels <input type="checkbox"/> Peer Partner Learning <input type="checkbox"/> Project team <input type="checkbox"/> Laboratory Groups <input type="checkbox"/> Think, Pair, Share <input type="checkbox"/> Cooperative Learning <input checked="" type="checkbox"/> Tutorial Groups <input type="checkbox"/> Interviewing <input type="checkbox"/> Conferencing <input type="checkbox"/> Other (List)

**Scope and Sequence:****Overarching Enduring Understandings for the course**

- Mathematics is a useful language for symbolically modeling and thus simplifying and analyzing our world.
- Mathematics is a logical and objective means of analyzing and solving problems.
- The effective communication of mathematics is essential to its application. [C4]

**Topical Enduring Understandings for the course**

- The concept of a limit is one of the foundations of calculus.
- The limit of a function is the value approached by  $( )fx$  as  $x$  approaches a given value or infinity.
- The derivative is the instantaneous rate of change at a given point.
- The integral is a function that can be used to determine the summation of an infinite set.
- Differentiation and definite integration are inverse operations.
- The definite integral can be used to find exact area, volume, or length by using the limit of Riemann sums.
- There is a defined relationship between the integral of function  $f$ , the function  $f$ , and the first and second derivatives of function  $f$ .
- Integrals can be used to solve a variety of problems related to area, velocity, acceleration, volume, area of a surface of revolution, length of a curve, and work.

**Unit 1—Review Precalculus Unit**

- Three weeks to complete.  
Two weeks to complete for BC students.
- Unit 2—Limits & Continuity**
- Three weeks to complete for AB students.
- Two weeks to complete for BC students.
- Unit 3—Derivative Basics**
- Four weeks to complete for AB students.
- Three weeks to complete for BC students.
- Unit 4—Derivatives with the Chain Rule**
- Three weeks to complete for AB students.
- Two weeks to complete for BC students.
- Unit 5—Applying Derivatives**
- Four weeks to complete for AB students.
- Three weeks to complete for BC students.
- Unit 6—Basics of Integrals**
- Four weeks to complete for AB students.
- Three weeks to complete for BC students.
- Unit 7—Solving Differential Equations**
- Four weeks to complete for AB students.
- Three weeks to complete for BC students.
- Unit 8—Application of Integrals**
- Three weeks to complete for AB students.
- Two weeks to complete for BC students.
- Unit 9--Series (BC students only)**
- Two weeks to complete for BC students.
- Unit 10—Polar & Vectors (BC students only)**
- Two weeks to complete for BC students.
- Review Unit—Practice taking AP Tests**
- One to two weeks to complete for AB students.
- One to two weeks to complete for BC students.

Adapted from Understanding by Design Template available online and the Understanding by Design: Professional Development Workbook.

References:

Wiggins, G., & McTighe, J. (n.d.). Understanding by Design Exchange. Retrieved November 2, 2004 from <http://www.ubdexchange.org/>

Wiggins, G., & McTighe, J. (2004). Understanding by design: Professional development workbook. Alexandria, VA: Association for Supervision and Curriculum Development.)